Time measurement: Criticism of a paper by L. M. Stephenson

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counts $\nu T' = (1-v^2/c^2)^{1/2}N$ signals from the proper clock and $\nu' T' = (1-v/c)N$ signals from the clock in S. The proper frequency is the same for both observers as the clocks are identical and in the same internal state; so $1/\nu$ can be used to define the unit of time by both of them. Obviously, the number of signals (or ticks) is not invariant. The difference between the number of signals from the proper clock and from the unproper clock is equal to

$$\left(1-\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right)N.$$

for the observer in S and to

$$\left(1-\frac{v^2}{c^2}\right)^{1/2}\left(1-\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right)N$$

for the observer in S′.† All these results agree with the usual interpretation of the time dilatation. There is not a single argument against the time dilatation.

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Time measurement: Criticism of a paper by L. M. Stephenson

Abstract. A recent paper on time measurement by L. M. Stephenson is examined in detail. It is argued that the paper sheds no new light on the subject and that it is altogether misleading. In particular, while the notion of a primary time scale in special relativity bears some relation to certain elements that are familiar and valid, Stephenson's own treatment of the notion appears to be wholly erroneous.

† Both frames are treated on an equal footing but the equations are not completely symmetrical to $v \to -v$, $S \to S'$, $S' \to S$. To acquire complete symmetry the discussion should be repeated starting from the pair of events ($x' = 0$, $t' = 0$) and ($x' = -vT'$, $t' = T'$) in S′.

Introduction

A recent paper on time measurement by Stephenson (1970) purports to disclose certain shortcomings in the standard interpretation of the theory of special relativity. He claims to take account of features that have been overlooked by all other writers One's attention has lately been called to this paper. Since it appears unfortunately to create much unnecessary confusion without supplying anything new whatsoever, one feels obliged to say so. Also, since Stephenson's discussion is very involved—and, one is bound to say, obscure—one is driven to examine it in regrettable detail.

Stephenson's contentions

There are three features of Stephenson's paper that, at any rate when they are taken together, make it ostensibly different from most other criticisms of special relativity.

(i) He implicitly accepts special relativity as both selfconsistent and as applicable to the actual physical world; apparently he maintains simply that the application made by other writers to the treatment of time measurement is incorrect.

(ii) He wishes to emphasize the property of a 'complete clock' of having two 'halves', one that oscillates or ticks, and one that keeps count of the oscillations or ticks.

(iii) He claims that special relativity predicts a 'universal primary time scale', by which he means, apparently, more than merely a *scale*, in fact a universal time as in classical physics.

Special relativity

In special relativity, as in classical mechanics, an inertial system of reference consists of a frame, that may be regarded as an actual rigid body, and a system of time keeping. According to the postulates, one such inertial system exists; the frame may be graduated in a prescribed manner to give coordinates x, y, z; to each material point of the frame there is attached a clock; all such clocks are identical, in the sense of having been constructed in accordance with a common specification; each clock is 'complete' in the sense of Stephenson (although he might say that a complete clock is not necessarily one of these standard clocks); all these clocks are synchronized by a specified procedure. These properties are not trivial but, as already stated, they follow from the postulates of the theory. It then further follows from these postulates that there exists a triple infinity of such inertial systems—all of the same status, each inertial frame being in uniform motion relative to any other, the motion of each being free and 'non-rotating'—and all the clocks in all such systems are standard clocks.

If E is any event and $\mathscr{I}$ is any inertial system, E occurs at a marked point $P(x, y, z)$ in the inertial frame and at a time t by the standard clock fixed at P. Then E may be said to be the event (x, y, z, t). If $\mathscr{I}'$ is any other inertial system, then 'coordinates' x', y', z', t' may be similarly defined so that E is also the event (x', y', z', t'). For any pair $\mathscr{I}, \mathscr{I}'$ the transformation from (x, y, z, t) to (x', y', z', t'), or vice versa, is called a Lorentz transformation. For any particular pair, the frames may be chosen so that the transformation takes the familiar form which is evidently employed, in particular, by Stephenson.

Time measurement

Nearly all criticisms of special relativity with regard to time measurement (such as Stephenson's) reduce to the comparison of two clocks alleged to 'measure' the time interval between the same given pair of distinct events, E, F, say. Now a clock *measures* a time interval between the two events only if it is present at each of E, F. If the interval EF in 4-space is space-like or null, no clock can be present at both E, F. If EF is time-like, one and only clock $\mathscr{C}$ fixed in an inertial frame is present at E, F; any number of other clocks can be present at E, F in this case, but if $\mathscr{C}^*$ is any one of these then $\mathscr{C}^*$ cannot move throughout with any single inertial frame. Consequently $\mathscr{C}$, $\mathscr{C}^*$ are not mutually symmetrical in their relation to E, F. So they measure different intervals between E, F; this is the usual so-called 'clock paradox'. Stephenson may divide each of the clocks $\mathscr{C}$, $\mathscr{C}^*$ into as many 'halves' as he will, but he cannot alter this result as a consequence of special relativity. And, as we have remarked, Stephenson apparently accepts this theory.

We may remark that the particular clock $\mathscr{C}$ measures the geodesic interval between E, F which, in the Minkowski geometry of special relativity is the longest—not the shortest—interval between two events. So $\mathscr{C}^*$ measures a shorter time interval than $\mathscr{C}$.

If any other clock $\mathscr{C}^\dagger$ is not present at both E and F it cannot be said to *measure* a time interval between E, F. Therefore, it is meaningless to assert that $\mathscr{C}^\dagger$ gives a result that is discrepant or accordant with any other clock.

Naturally, an observer attached to the clock $\mathscr{C}^\dagger$ may observe the occurrence of the events E, F by means of light signals emitted at E, F. But he would obviously not call the time interval between his observations the time interval between E, F. For he would have to make allowance for the 'times' taken by the signals to reach $\mathscr{C}^\dagger$ from E, F. However, when he has done this, according to some standard procedure, he has not measured an interval EF by any one particular clock.

When it is stated as a consequence of special relativity that a moving clock appears to go 'slow', this is a loose statement of a very precise result. It is that any one particular clock $\mathscr{C}'$ say, fixed in a frame in a system such as $\mathscr{I}'$ and reading time t', moves past a succession of the synchronized standard clocks fixed in the frame in the system $\mathscr{I}$, and if when $\mathscr{C}'$ reads t_1', t_2', it passes clocks in $\mathscr{I}$ reading t_1, t_2, then

$$t_2' - t_1' = (1 - v^2/c^2)^{1/2}(t_2 - t_1). \qquad (1)$$

This is a perfectly well-defined observable result, but it relates two readings of one clock to one reading of each of two other clocks. It is not the comparison of *a* moving clock with *a* stationary clock.

Stephenson's examples

Stephenson's first example is that of an observer O who is stationary in one inertial frame $\mathscr{F}$ say, and an observer A who moves past O with uniform velocity v along the x axis to a point distant L_0 from O in frame $\mathscr{F}$; the motion of A is reversed at this point and A subsequently re-passes O with uniform velocity $-v$ along Ox. As Stephenson says, this is an elementary example of the so-called clock paradox. But he claims that 'the usual analysis does not relate to the reading of a *complete* clock'. Stephenson supposes that the question of the acceleration concerned in reversing the motion of A is disposed of in the usual way, and he admits that this causes no difficulty.

We continue to deal with standard clocks. When A's clock is passing a clock fixed in $\mathscr{F}$ let t', t be the readings of the two clocks, respectively. Let the clocks be adjusted so that $t' = 0$, $t = 0$ refer to the event of A first passing O. Then equation (1) is equivalent to

$$t' = (1 - v^2/c^2)^{1/2}\, t \qquad (2)$$

and this again gives the result expressed by saying that the moving clock appears to go slow. At A's turn-round, $t = L_0/v$ and so $t' = (1-v^2/c^2)^{1/2} L_0/c$. A's return journey can be treated similarly. Therefore when A re-passes O, A's clock reads T_1 and O's clock reads T_2, say, where

$$T_1 = 2(1-v^2/c^2)^{1/2} L_0/v, \qquad T_2 = 2L_0/v. \tag{3}$$

These are precisely the results S(3), S(5). (I refer to Stephenson's § 1 as S1, his equation (1) as S(1), and so on.) Stephenson describes T_2 in the same terms as we do. But he appears to describe T_1 in different terms; he says that O " 'observes' that the total 'time' elapsed on A's 'clock' " is T_1. However, the whole point of the comparison is that T_1 is the total time elapsed on A's clock, and T_2 the total time elapsed on O's clock, between the two encounters, *as observed by all observers*. Stephenson seems to make a special point of the fact that two observers agree about the readings of a clock carried between two particular events, but this is entirely trivial. The so-called clock paradox applies to a difference between the readings of two different clocks carried between the same two events, and the result is significant precisely because *all observers agree about it*.

Incidentally, Stephenson's own derivation of S(3), S(5) is not clear because, after taking other writers to task for not adequately defining their terms, he himself gives no definition at all of the quantities $\Delta\tau_a$, $\Delta\tau_0$ that appear in his very first equation!

What we have just said about the agreement between observers appears to be the only point brought out in Stephenson's next calculation in S4.1. This is a roundabout way of calculating the number of periods of A's clock observed by O to occur between the two encounters of O, A previously considered. He reckons this to yield the time T_3 in S(8). But, as we have remarked, this number of periods is trivially the same for all who observe it, and so naturally $T_1 = T_3$, as Stephenson verifies.

Precisely the same comment applies to the result S(10) in Stephenson's next example in S4.2, where an observer fixed in a different position in $\mathscr{F}$ is supposed to observe A's clock. He then asserts that in the two examples S4.1, S4.2, O's clock will 'read' a time T_5 given by S(12). The case of S4.1 is the same as that yielding the time T_2 and so, naturally, $T_2 = T_5$. The case S4.2 is not the same because the new O no longer experiences the events concerned. But the new O is fixed in the same inertial frame as the old O; so if the new O observes at time interval T_5 two events that occur at the old O at time interval T_2, then again $T_2 = T_5$. (Note that this is not the same as the case previously mentioned of the clock $\mathscr{C}^\dagger$ and the events E, F. In the case now considered, we have two observers that are given as being fixed in the same inertial frame and we have two events that occur at a single point in this frame. Obviously the time interval between observing the two events must be the same for these two observers.) Actually Stephenson himself is not explicit about the events that the new O is supposed to observe on his clock.

In S4.3 Stephenson writes: 'As $T_1 = T_3 = T_4$ the *realizable experiment* appropriate to the usually quoted result corresponds to having the first half of an atomic clock in the moving frame of reference and the second half in the observer's frame of reference'. I repeat that if A's clock reads $t' = 0$ at one event and $t' = T_1$ at another event, then (trivially) *all* observers must agree that this is the case and so (trivially) $T_1 = T_3 = T_4$; the 'realizable experiment' is for *anyone* to read the clock.

Stephenson's S5 appears to be mathematically wrong. He is dealing with observers O, A as in his first example; O reckons that A travels away from O with speed v to distance L_0 and A then reverses. At the event of reversal, A's clock reads

$(1-v^2/c^2)^{1/2} L_0/v$, as we have said. So A reckons that O has travelled away from A the distance $(1-v^2/c^2)^{1/2} L_0$ with speed v, *not* the distance L_a in S(14). This is a feature of the asymmetry between O, A that shows the clock 'paradox' to be no paradox (see McCrea 1951, Crampin *et al.* 1959).

Stephenson then states: 'We may thus conclude that the reading of A's clock read-out indicator, as observed by O, will always be the same as the reading of the indicator of an *identical* clock situated in O's frame of reference'. This proposition is not valid by virtue of the work just preceding it because, as we have just seen, this work is incorrect. But anyhow, on account of its use of the phrase 'always', the proposition either has no physical significance or else it pre-supposes the existence of universal time, which it purports to establish.

The latter part of S5 (Stephenson 1970 p. 374) concludes with statements about 'the primary time scale, *which directly gives the time elapse between events*' and which is 'affected by both the time *and length* transformations of special relativity'. Presumably he means by this that the effects of the two transformations cancel each other out, but as we have seen, this conclusion is based on his erroneous length transformation S(14). He goes on to assert 'any given complete clock will measure the same time elapse between two events in all inertial frames of reference'; I have already pointed out that a clock fixed in at most one inertial frame can be present at each of two events, and so the phrase 'in all inertial frames of reference' is inapplicable or irrelevant. Any number of other clocks not fixed in any one inertial frame may be present at the two events; the time elapse measured by any of these clocks will be, in general, different from that measured by any other. Stephenson says nothing that reveals any fallacy in this standard result of special relativity.

Stephenson then has S6 on "The apparent 'lifetime' of μ-mesons". A conclusion does not emerge. But regarding 'the average distance travelled by a μ-meson, situated in frame A, as observed by a stationary observer O', he states: 'The apparent path length travelled by μ-mesons is thus increased by the factor of $(1-v^2/c^2)^{-1/2}$... '. The only interpretation one can place upon the description is that A is the rest-frame of the meson and consequently that the path length in A is zero; so it is not evident what 'apparent path length' is increased.

Thus it is seen that a patient examination of Stephenson's paper detects no proof of any result at variance with standard special relativity theory. When Stephenson writes in S7 of 'sources of error' he appears to be referring to the need to distinguish between what an observer actually sees in a certain situation and what he finds when he 'plots' or 'reduces' his observations in a prescribed manner. But this is a distinction that is as old as physics itself. It is nothing peculiar to relativity theory, although serious writers on relativity are in fact specially careful about it—they have to be because the light-propagation involved in the 'seeing' is so basic to the whole subject.

Primary time scale

In spite of these strictures, it can be admitted that there are three valid general features of time measurement that Stephenson seeks to expose. These are (*a*) the possibility of an agreed common time-keeping for all observers, (*b*) the possibility of setting this up while treating all inertial observers as being on the same footing, and (*c*) a possible 'Machian' element in time-keeping. In discussing Stephenson's paper it seems essential to try to elucidate these features. They are not new, and I believe Stephenson's own arguments about them to be false and his implied criticisms of special relativity to be invalid. We consider the features in turn.

(*a*) It is necessary to say a word about time-keeping in classical physics. The classical model of the physical world admits universal time in the following sense. We may suppose there are any number of identical clocks at every event. If E, F are any two events, then clocks may be transported from E to F and not from F to E, when F is said to be after E, or vice versa, when E is said to be after F, or neither, when E, F are said to be simultaneous. If clocks can be transported from E to F then, in whatever way this is done, they will all agree at F. If clocks can be transported from E to F and from F to G, then they can be transported from E to G.

Special relativity physics denies the existence of universal time in this sense because it constructs a model of the physical world in which the setting up of such a time is *operationally impossible*.

Now everyone knows that we can never claim that the actual physical world is bound to behave in accordance with any mathematical model. But we can, of course, know that the physical world does *not* behave in accordance with some given model. Most physicists are convinced that it does not behave in accordance with classical physics, in particular, with regard to universal time. This is because they understand that the postulated existence of the operational means of setting up universal time contradicts experience. This 'operational' approach to physics was that adopted by Einstein in formulating special relativity, and it is now, of course, familiar throughout physics; there can be no going back upon it.

While there cannot be universal time in the classical sense, in any model physicists if they wish may set up a 'primary time scale' according to some agreed arbitrary convention, but one that is operationally realizable.

In the present communication we are concerned with the special relativity model. We have recalled how, in any inertial system $\mathscr{I}$, we postulate the existence of a standard clock fixed at every point of the frame. Then in $\mathscr{I}$ every event E has in particular a coordinate t that has a precise operational meaning; it is the reading of the standard clock in $\mathscr{I}$ that is present at E. It does not matter what observer reads the clock. So all observers could agree to use this as their primary time scale. That is to say, of all the equivalent inertial systems, the observers could arbitrarily select one ($\mathscr{I}$) and declare that the official time of any event E is to be the reading of the $\mathscr{I}$-clock present at E. In general, this would be a useless and inconvenient procedure! Also, of course, the selection of a different inertial system would give a different time that would be no better and no worse than the first. We simply have an illustration of the fact that special relativity does not prevent observers using a common time scale if they want to do so; they can call it their primary time scale, again if they want to do so.

(*b*) It may next be asked whether special relativity admits any such time scale that gives no preference to any one inertial system. Now if a clock can be present at each of two events E, F we have seen that there is one and only one standard clock fixed in an inertial frame that is present at each. The time elapse measured by *this* clock is, of course, a quantity $\tau(E, F)$, say, about which all observers agree. If then E is some arbitrarily chosen particular event, and if we write $\tau = \tau(E, F)$ provided F is such that this (real) quantity exists, then τ may be treated as providing a primary time scale for events like F. Actually, τ is real for any event F inside the null cone of E, and τ may be taken positive if F is in the 'forward' half-cone and negative if F is in the 'backward' half-cone.

If (x, y, z, t) are the coordinates of F in any inertial system $\mathscr{I}$ in which E has been chosen as origin, then

$$c^2\tau^2 = c^2t^2 - x^2 - y^2 - z^2.$$

Obviously this is an invariant for all systems $\mathscr{I}$, and the existence of τ is trivial when looked at in this way. It was introduced as above, partly in order to show its operational character, and partly because it is seen to be apparently the nearest thing possible to Stephenson's 'primary time scale'. But it is now seen to be far more arbitrary than Stephenson implies. If we take any different particular event E' as origin, we get a different time τ', and not every event that has a real τ has a real τ', and vice versa.

(*c*) The Machian element is closely bound up with the notion of cosmic time and this we now briefly discuss.

Cosmic time

In the standard development of special relativity there is no explicit mention of any general material contents of the model. If there are such contents, and if their behaviour is not wholly chaotic, and if we still work within the postulates of special relativity, then any event can be taken to be at the origin O of an inertial frame that is related in some special way to the material. For instance, O may be moving with, say, the mean motion of the material in its vicinity; or O may be moving so that, say, the resultant radial motion of remote material is zero. So in some such sense there may be a preferred inertial frame at each event.

A particular case of this is the special relativity, or 'Milne', cosmological model. This is the case where the world-lines of all fundamental observers pass through a unique event E (the 'big-bang') and every observer sees the same picture as every other. In this case, if E is chosen to be the particular event in the definition of the time τ above, then τ is the *cosmic time* of the model. (Kermack and McCrea 1933.)

These considerations are mentioned here in order to show that it is only when we take explicit account of the material present that we may begin to single out a particular preferred primary time scale. It may indeed be natural to do this, and this may be a clue to what Stephenson calls 'Machian' relativity at the end of his paper.

However that may be, so far as the theory of special relativity itself is concerned, Stephenson's work appears to be unfortunately and entirely misleading.

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Impact ionization of micro-particles at low velocities

Abstract. Impact ionization of micrometre-size particles has been observed at velocities down to 60 m s^{-1}, where the empirical law $Q \propto m^{\alpha}v^{\beta}$ is shown to remain valid, the indices having the values $\alpha = 1$ and $\beta = 3{\cdot}4$.

The impact of micrometre-size metallic particles on a molybdenum target has been shown to produce substantial ionization at velocities as low as 60 m s^{-1}, and the